Name: ______________________________________ Date: ___________________

A2 CC: Horizontal Stretching of Functions

Warm Up:

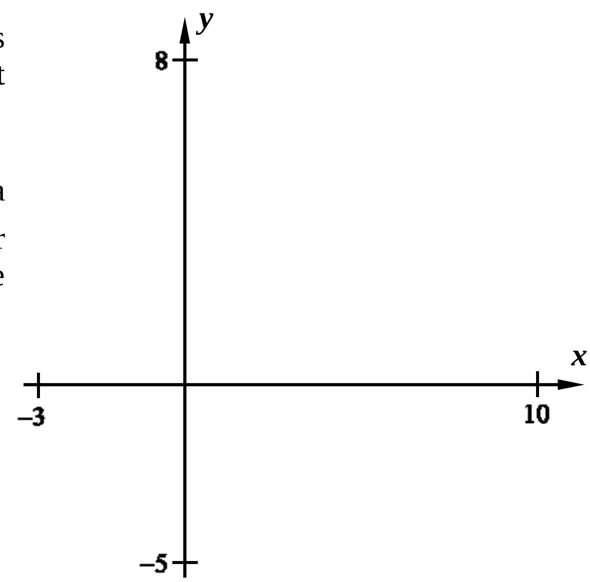
The function h(x) has a range given by the interval [-2, 8]. The function f(x) is defined by $f(x)=\frac{1}{2}h(x)+6$.

What is the range of f(x)?

Perhaps one of the hardest transformations of functions occurs when we horizontally stretch and compress the function. Yet these types of transformations can be vital, especially when we model a process over time and then "shrink" the time interval.

***Exercise* #1:** Consider the absolute value function $f(x)=|x-2|-3$.

(a) Using your calculator, sketch a graph of *f* on the axes provided. Label the coordinates of its vertex point without the use of your calculator.



(b) Consider the function $g(x)=f(2x)$. Determine a formula for *g* and then graph it on the axes. Use your calculator to find its minimum point and label it on the graph.

(c) Now consider the function $h(x)=f\left(\frac{1}{2}x\right)$. Determine a formula for *h* and graph it on the axes. Use your calculator to find its minimum point and label it on the graph.

(d) Summarize your findings below for each function.

$f(x)$ turning point: $f(2x)$ turning point: $f\left(\frac{1}{2}x\right)$ turning point:

(e) What stayed constant about the turning points? What changed and how did it change?

This one exercise shows a remarkable, and counterintuitive, concept about horizontal dilations:

HORIZONTAL DILATIONS

For a real number, positive constant such that $k > 1$:

1. The function $f(kx)$ represents a horizontal compression of $f(x)$ by a factor of k

2. The function $f\left(\frac{1}{k}x\right)$ represents a horizontal stretch of $f(x)$ by a factor of k.

***Exercise* #2:** Let's take a look at the **quadratic function** $f(x) = x^2 - 12x + 20$.

(a) Determine the coordinates of its turning point by using the equation for the axis of symmetry of $x = -\frac{b}{2a}$.

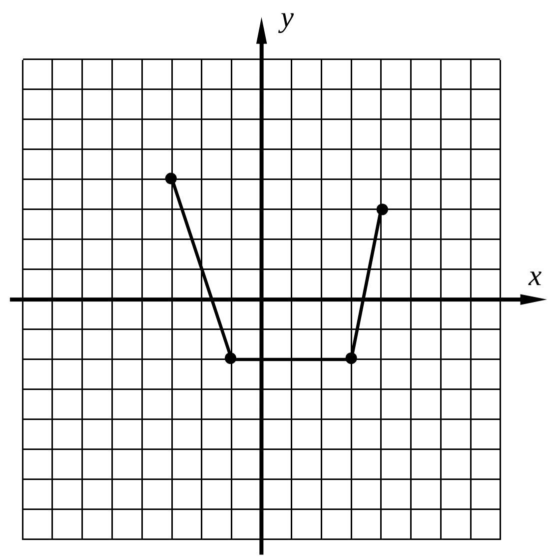
(b) If g is defined by $g(x) = f(3x)$, what should be the coordinates of its turning point based on our previous work? Explain.

(c) Determine a formula for $g(x)$ and then use the turning point formula to verify your answer from part (b).

(d) Show that the y-intercept of both $f(x)$ and $g(x)$ are equal. What does this make sense from a horizontal dilation perspective?

We can truly investigate why this is happening by looking at a function that is only represented graphically.

***Exercise* #3:** Consider the function $f(x)$ graphed on the grid below. If $g(x) = f\left(\frac{1}{2}x\right)$ for all values of x then answer the following questions.



(a) Evaluate each of the following using the definition of g and then state the point that lies on its graph as a consequence.

$g(-6) =$

$g(-2) =$

$g(6) =$

$g(8) =$

(b) Graph g on the grid to the right. How would you describe its graph compared to the graph of $f(x)$?

HOMEWORK

1. The quadratic function $g(x)$ has a turning point at $(-12, 8)$. Where would the quadratic function $f(x) = g(4x)$ have a turning point?

(1) $(-48, 32)$ (3) $(-3, 8)$

(2) $(-48, 8)$ (4) $(-3, 2)$

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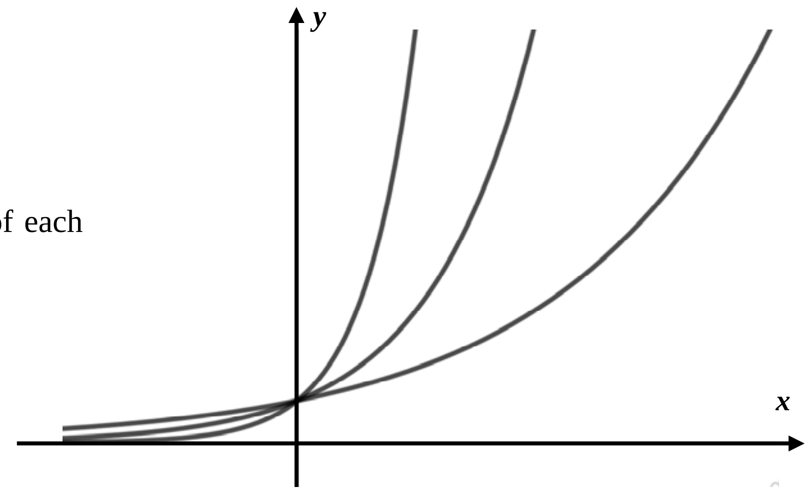
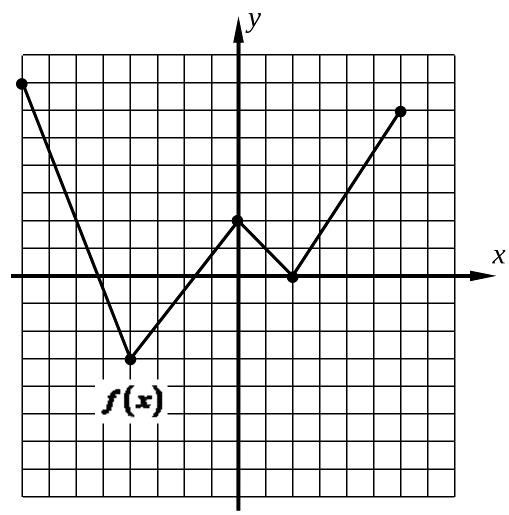
2. The three exponential graphs shown below represent the function $f(x) = b^x$, $g(x) = b^{2x}$ and $h(x) = b^{\frac{x}{2}}$, for some $b > 1$.

(a) Label each with its correct equation.

(b) Algebraically, show that the y-intercept of each function is the same.



3. The graph of $f(x)$ is shown on the grid below. Sketch a graph of $f(2x)$ on the same set of axes.



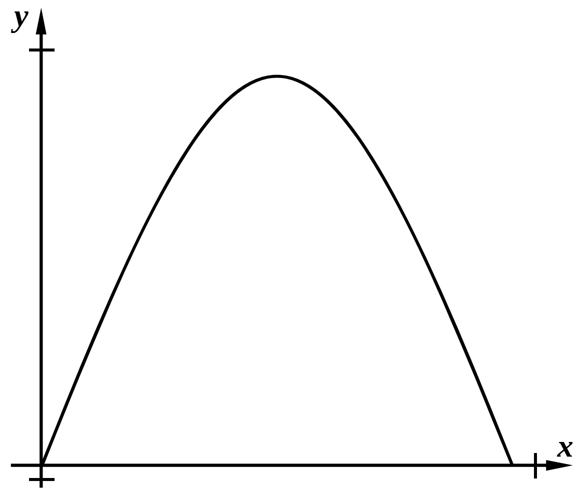
State the domain of the two functions:

Domain of $f(x)$: Domain of $f(2x)$:

APPLICATIONS

4. An arch is to be constructed so that its shape follows the curve $y=-\frac{1}{2}x^2+10x$, where x measures the horizontal distance along the ground and y measures the vertical height of the arch above the ground, both in units of feet. The general graph of this arch is shown below.

   (a) Based on this equation, what is the height of the arch at the turning point? Show the work that leads to your answer.



   (b) If a second arch was to be created that had the same height, but only half the width, determine an equation for this arch based on our work in this lesson.

   (c) Choosing an appropriate graphing window based on (a), graph the second arch on the axes above. Label your graphing window. Use your calculator to determine the new turning point and label both points on the graphs.

REASONING

5. We've seen repeatedly that a horizontal dilation does not alter the graph's y-intercept. Given the function $f(x)$ and $g(x)=f(kx)$, can you determine an algebraic argument for why $f(x)$ and $g(x)$ must have the same y-intercepts? (Hint: Think about how we **always** find the y-intercept of any function).

6. If the function $f(x)$ has a domain of $-2\le x\le 8$ and a range of $-4\le y\le 6$ and the function $g(x)$ is defined by the formula $g(x)=5f(2x)$ then what are the domain and range of g? Explain your thought process.